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TWO-DIMENSIONAL LAMINAR BOUNDARY LAYER IN FLOW OF THERMODYNAMICALLY EQUILIBRATED WATER VAPOR

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This paper examines the boundary problem of a laminar boundary layer in flow of thermodynamically equilibrated water vapor. An approximate method of solution is proposed, based on an approximation for the density and the coefficient of dynamic viscosity across the layer.

At present only experimental investigations are known of flow in a boundary layer formed in a nozzle with motion of water vapor [1, 2]. In particular, it is noted that the boundary layer in a Laval nozzle, when it has spontaneous condensation, when the liquid phase is finally dispersed, consists of two sublayers: an upper vapor-drop layer and a lower heated vapor layer (adjacent to the wall).

Below we present results of a theoretical investigation of flow in a two-dimensional laminar boundary layer with motion of water vapor along a solid impermeable surface.

Statement of the Problem. For the water vapor we assume that the liquid phase is in a finely dispersed state and that the velocities of the phases are the same: the thermodynamic equilibrium of the phases is not perturbed, and the temperature and pressure obey the vapor pressure curve. With these assumptions water vapor can be regarded as some kind of imperfect gas.

In considering the equations of motion and continuity of the laminar boundary layer, derived, e.g., in [3], no assumptions of any kind are made concerning the gas being perfect. Therefore, these equations will be valid in our case.

Having made the transformations usually applied in boundary layer theory, we obtain the energy equation for a two-dimensional steady-state laminar boundary layer in an imperfect gas

$$\rho u \frac{\partial h}{\partial x} - \rho v \frac{\partial h}{\partial y} = u \frac{dp}{dx} + \mu \left(\frac{\partial u}{\partial y} \right)^2 + \frac{\partial}{\partial y} \left(\lambda \frac{\partial T}{\partial y} \right).$$

To determine the dynamic viscosity of a two-phase medium we use the "Einstein correction" [3]:

$$\frac{\mu}{\mu_v} = 1 - m \frac{\mu_l + 2,5\mu_l}{\mu_v + \mu_v},$$

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where m is the volume fraction of the liquid phase, equal to $m = [1 + (z/(1-z))V_v/V_l]^{-1}$. Since $V_v \gg V_l$ for flows with a large degree of dryness we obtain the result that $m \ll 1$ and $\mu \cong \mu_v$. For example, for water vapor with $z \geq 0.7$, the error at T = 373°K is 0.07%, and at T = 273°K it is 0.0005%.

Later we assume that $\mu = \mu_v$, and that the dynamic viscosity of the vapor phase is a linear function of temperature.

The temperature and pressure of water vapor in thermodynamic equilibrium are connected by the single-valued relation $p = f(T)$. Since $\partial T/\partial y = 0$ for a boundary layer, in the water vapor case $\partial T/\partial y = 0$. Hence it follows that $T = T_e(x)$ and $\mu = \mu_e(x)$.

For a high degree of dryness $(1-z)V_l \ll zV_v$, and the Clapeyron–Clausius equation reduces to the form

$$\rho(h - h_l) = T\frac{dp}{dT},$$

where $h_l(T)$ is a known function of temperature.

In solving the problem of a laminar boundary layer in flow of water vapor in the general case, one must take into account that a region of single-phase state (heated vapor) is formed near a solid surface [1, 2]. For a heated vapor we assume a model of a perfect gas with linear viscosity law.

Taking this into account, we write the systems of equations for the regions of moist and heated vapor, respectively:

$$\left.\begin{aligned} &\rho u\frac{\partial u}{\partial x} + \rho v\frac{\partial u}{\partial y} = -\frac{dp}{dx} + \mu_e\frac{\partial^2 u}{\partial y^2}, \quad \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0,\\ &\rho u\frac{\partial h}{\partial x} + \rho v\frac{\partial h}{\partial y} = u\frac{dp}{dx} + \mu_e\left(\frac{\partial u}{\partial y}\right)^2, \quad \rho(h - h_l) = T\frac{dp}{dT}, \quad p = f(T),\end{aligned}\right\} \tag{1}$$

$$\left.\begin{aligned} &\rho u\frac{\partial u}{\partial x} + \rho v\frac{\partial u}{\partial y} = -\frac{dp}{dx} + \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right), \quad \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0,\\ &\rho u\frac{\partial h}{\partial x} + \rho v\frac{\partial h}{\partial y} = u\frac{dp}{dx} + \mu\left(\frac{\partial u}{\partial y}\right)^2 + \frac{1}{\Pr}\frac{\partial}{\partial y}\left(\mu\frac{\partial h}{\partial y}\right),\\ &\rho(h - h_c) = \frac{\varkappa}{\varkappa - 1}p, \quad \mu = ch + \mu_c,\end{aligned}\right\} \tag{2}$$

where h_c, c, μ_c are constants.

We can write the boundary conditions: at $y = 0$ $u = v = 0$, $h = h_T(x)$ or $\partial h/\partial y = 0$; for $y = \infty$ $u = u_e(x)$, $h = h_e(x)$. The second equation for the enthalpy corresponds to the case where the surface is thermally insulated.

To these boundary conditions one must add the condition that the equation which we denote by $y = y_b(x)$ must be matched at the boundary; at $y = y_b(x)$ $u_1 = u_2$, $(\partial u/\partial y)_1 = (\partial u/\partial y)_2$, $v_1 = v_2$, $h_1 = h_2 = h_v$, subscripts 1 and 2 denote values on the water vapor and heated vapor sides, respectively. The enthalpy h_v at the boundary of the regions corresponds to the enthalpy at the saturation line and is a known function of T_e.

The initial conditions are: for $x = x_0$ $u = u_0(y)$, $h = h_0(y)$.

From the condition $h_0(y_{b0}) = h_v$ we determine the initial ordinate of the boundary of the regions $y_{b0} = y_b(x_0)$ and the values of the quantities required in it.

We note that from the second, third, and fourth equations of system (1) we can obtain the following final relation for the enthalpy:

$$h = \rho_e(h_e - h_l)\left\{\left[\mu_e\left(\frac{\partial u}{\partial y}\right)^2 - T\frac{dp}{dT}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)\right]/uh_l'(T)\frac{dT}{dx} - \frac{T}{h_l'(T)}\frac{d^2p}{dT^2}\right\}^{-1} + h_l. \tag{3}$$

From what has been said above, it can be seen that there is considerable difficulty in solving the laminar boundary layer with flow of water vapor. It should be noted that the boundary of the regions and the values of the parameters are unknown and are determined during the solution.

Approximate Method of Solution. It follows from the density expression for water vapor [$\rho = 1/zV_v$, $(1-z)V_l \ll zV_v$] that the variation along an isobar is determined by the dryness level z, since for p = const V_v = const. In the boundary layer $z_e \leq z \leq 1$. For large values of z_e (≈ 0.8) the density varies only a little across the boundary layer in the water vapor region.

For the heated vapor region it follows from the Clapeyron–Mendeleev equation that the density variation is determined as a function of temperature. If the parameters for stagnation of the outer flow are near the top boundary curve and there is no strong heating of the surface, then the variation of temperature, and therefore of density, in the heated vapor region will be very little.

To sum up, we can say that for these cases (e.g., for the working nozzles of water vapor ejector pumps and for steam turbines) we can neglect density variation across the boundary layer. This refers also to the dynamic viscosity coefficient, which varies only in the heated vapor region for p = const.

To obtain an approximate solution, we assume the density and the dynamic viscosity coefficient to be constant and equal to unity across the boundary layer for both regions $\rho \approx (1+\varepsilon_1)\rho_e$, $\mu \approx (1+\varepsilon_2)\mu_e$ (ε_1, ε_2 are constant and quite small compared with unity). Then the equations of motion and continuity for the two regions take the form

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_e u_e' + \nu_e a\frac{\partial^2 u}{\partial y^2}, \quad \frac{\partial(\rho_e u)}{\partial x} + \frac{\partial(\rho_e v)}{\partial y} = 0, \quad a = \frac{1+\varepsilon_2}{1+\varepsilon_1}. \tag{4}$$

To this system of equations we apply the transformation

$$\bar{x} = a\int_{x_0}^{x} \nu_e(\zeta)\rho_e^2(\zeta)\,d\zeta, \quad \bar{y} = \rho_e(x)\,y, \quad \bar{u} = u, \quad \bar{v} = \frac{v}{a\nu_e\rho_e} + \frac{\rho_e'}{a\nu_e\rho_e^2}\,yu, \quad \bar{u}_e = u_e$$

(the primes denote derivatives with respect to x). We obtain equations which correspond completely to the equations of two-dimensional flow of an incompressible liquid:

$$\bar{u}\frac{\partial\bar{u}}{\partial\bar{x}} - \bar{v}\frac{\partial\bar{u}}{\partial\bar{y}} = \bar{u}_e\frac{d\bar{u}_e}{d\bar{x}} + \frac{\partial^2\bar{u}}{\partial\bar{y}^2}, \quad \frac{\partial\bar{u}}{\partial\bar{x}} + \frac{\partial\bar{v}}{\partial\bar{y}} = 0. \tag{5}$$

The boundary conditions are:

$$\text{for } \bar{y}=0 \ \bar{u}=\bar{v}=0, \quad \text{for } \bar{y}=\infty \ \bar{u}=\bar{u}_e, \quad \text{for } \bar{x}=0 \ \bar{u}=\bar{u}_0(\bar{y}). \tag{6}$$

To solve the boundary problem of Eqs. (5) and (6) we can use the well-known methods developed for an incompressible liquid [3-6].

After solving the problem we find the enthalpy distribution in the boundary layer.

For the enthalpy in the water vapor region we can use Eq. (3). If we assume for the saturated liquid that the heat capacity c_l along the boundary curve is constant, and use the second equation of Eq. (4), we can simplify Eq. (3) after some transformations:

$$h = (h_e - h_l)\left[1 - \frac{\nu_e}{c_l}\frac{1}{u}\left(\frac{\partial u}{\partial y}\right)^2 \Big/ \frac{dT}{dx}\right]^{-1} - h_l \equiv F(x,\,y) \quad (y \geqslant y_b).$$

Now we return to the heated vapor region. Using the equations of continuity and state we can write the energy equation of system (2) in the form

$$\frac{1}{\Pr}\frac{\partial}{\partial y}\left(\mu\frac{\partial h}{\partial y}\right) - \mu\left(\frac{\partial u}{\partial y}\right)^2 = \frac{\varkappa p}{\varkappa-1}\left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\right) - \frac{u}{\varkappa-1}\frac{dp}{dx}. \tag{7}$$

Using the above approximation for the density and coefficient of dynamic viscosity, from Eq. (7) we obtain

$$\frac{\partial^2 h}{\partial y^2} = \Pr\left[A(x)\,u - \left(\frac{\partial u}{\partial y}\right)^2\right] \equiv R(x,\,y), \quad A(x) = \frac{p}{\mu_e(1-\varepsilon_2)(\varkappa-1)}\frac{d}{dx}\left(\ln\frac{p}{\rho_e^{\varkappa}}\right). \tag{8}$$

We now present the results of integrating Eq. (8), allowing for the matching conditions:

a) for a thermally insulated surface (for y = 0 $\partial h/\partial y = 0$)

$$h = h_v - \int_y^{y_b} \int_0^y R(x, y)\, dy dy, \quad 0 \leqslant y \leqslant y_b;$$

b) for a given enthalpy at the wall [for $y = 0$ $h = h_T(x)$]

$$h = h_T + \frac{y}{y_b}\left[h_v - h_T - \int_0^{y_b}\int_0^y R(x, y)\, dy dy\right] + \int_0^y \int_0^y R(x, y)\, dy dy,$$

$$0 \leqslant y \leqslant y_b.$$

For both cases y_b is found from the equation $F(x, y_b) = h_v$.

For applications of the boundary layer equation it proves very efficient to use similarity solutions. For system of equations (4) we analyze the distribution laws for parameters of the external flow, where the equations and partial derivatives reduce to ordinary differential equations, i.e., similarity exists.

From the continuity equations $\rho_e u = \partial\psi/\partial y$, $\rho_c v = -\partial\psi/\partial x$. We seek a stream function ψ in the form $\psi = k(x)\rho_e u_e \varphi(\eta)$, where $\eta = y/k(x)$, and $k(x)$ is an arbitrary function whose form will be determined later. Carrying out the transformation, we have

$$\varphi''' + \frac{\gamma(x)}{a}\varphi\varphi'' + \frac{\beta(x)}{a}(1 - \varphi'^2) = 0, \quad \varphi(0) = \varphi'(0) = 0, \quad \varphi'(\infty) = 1. \tag{9}$$

Here $\varphi'(\eta) = u/u_e$, $\gamma(x) = (k\rho_e u_e)'\, k/\mu_e$, $\beta(x) = u_e' k^2/\nu_e$, and the primes indicate derivatives with respect to the corresponding arguments.

The differential equation will be an ordinary one if $\gamma(x) = \text{const}$, $\beta(x) = \text{const}$. From the conditions $\gamma(x) = \text{const}$, $\beta(x) = \text{const}$ we obtain the form of the function $k(x)$ and the law for variation of the outer flow parameters in the implicit form

$$k(x) = \frac{c_1 u_e^{\frac{1}{\alpha}-1}}{\rho_e}, \quad x = c_2 \int \frac{u_e^{2\left(\frac{1}{\alpha}-1\right)}}{\rho_e \mu_e}\, du_e + c_3,$$

where $\alpha = \beta/\gamma$; c_1, c_2, and c_3 are constants.

In Eq. (9) we transformed to the new variables $\Phi = \sqrt{\gamma/a}\,\varphi$, $\xi = \sqrt{\gamma/a}\,\eta$. We obtain

$$\Phi''' + \Phi\Phi'' + \alpha(1 - \Phi'^2) = 0, \quad \Phi(0) = \Phi'(0) = 0, \quad \Phi'(\infty) = 1. \tag{10}$$

Equation (10) has been investigated by several authors over a wide range of variation of α, and tables of values of $\Phi'(\xi) = u/u_e$ $(\xi = y\sqrt{u_e'/\alpha\nu_e a})$ are presented, e.g., in [3].

We will consider the boundary layer on a semi-infinite flat plate in a homogeneous flow ($dp/dx = 0$). The boundary problem for Eq. (4) (without the initial condition) in this case reduces to the Blasius boundary problem concerning the boundary layer on a flat plate in flow of an incompressible liquid

$$2\varphi''' + \varphi\varphi'' = 0, \quad \varphi(0) = \varphi'(0) = 0, \quad \varphi'(\infty) = 1 \left(\frac{u}{u_e} = \varphi'(\eta),\right.$$
$$\left.\eta = y\sqrt{\frac{u_e}{x\nu_e a}}\right), \tag{11}$$

and its solution is derived, e.g., in [4].

Following elementary transformations and integration of the energy equation we obtain the enthalpy distribution in the water vapor region

$$\Theta(\eta) = 1 + 2\tau\int_\eta^\infty \frac{\varphi''^2}{\varphi}\, d\eta, \quad \eta \geqslant \eta_b, \quad \Theta(\eta) = \frac{h}{h_e}, \quad \tau = \frac{u_e^2}{h_e}. \tag{12}$$

The boundary of the region is determined from the equation $\int_{\eta_b}^\infty (\varphi''^2/\varphi)\, d\eta = (h_v/h_e - 1)/2\tau$.

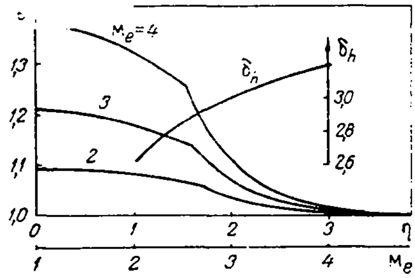

Fig. 1. The enthalpy distribution in the boundary layer of water vapor on a thermally insulated flat plate, and the dimensionless thickness of the enthalpy layer δ_h as a function of Mach number M_e.

For the heated vapor we assume that the Prandtl number is unity (this is valid, e.g., for water vapor over a wide range of variation of parameters [7]), and then there is evidently a particular integral of the energy equation [3]

$$h + \frac{u^2}{2} = b_1 u - b_2 \quad \text{or} \quad \Theta(\eta) = n_2 - n_1\varphi' - \frac{\tau}{2}\varphi'^2, \quad \eta_b \geqslant \eta \geqslant 0.$$

The constants n_1, n_2 (or b_1, b_2) are determined from the matching condition and the boundary condition at the wall.

Figure 1 shows results of calculating the enthalpy in the boundary layer in flow of water vapor along a thermally insulated flat plate for various values of Mach number M_e and constant stagnation parameters of the external flow $p_e^* = 5$ Bar, $T_e^* = 473.15$°K. The same figure shows the dimensionless thickness of the enthalpy layer δ_h as a function of Mach number. The thickness of the enthalpy layer is assumed to be the value where the difference in the enthalpy from that of the external flow is 1%.

The dimensionless thickness of the dynamic layer, determined from the condition that the velocity should differ by 1% from u_e, is 5.0 [4]. It is clear that the enthalpy layer thickness is less than the dynamic layer thickness.

In Eq. (12) there is the improper integral of the form

$$\int_b^\infty \frac{\varphi''^2}{\varphi}\, d\eta = E(b). \qquad (13)$$

We will show that it converges for b > 0. We write Eq. (11) in the form $d\varphi''/\varphi'' = (1/2)\varphi d\eta$, whence $\ln(\varphi''/\varphi''(b)) = -(1/2)\int_b^\eta \varphi d\eta$. Using the last relation and converting in the integral to the new variable $\varphi(d\eta = d\varphi/\varphi')$ and taking into account that $\varphi' \leq 1$, we obtain the estimate that $\varphi'' \leq r\exp(-\varphi^2/4)$, r = const. Now $E(b) \leq r\int_b^\infty (1/\varphi)\exp(-\varphi^2/2)d\eta = r\int_{\varphi(b)}^\infty 1/\varphi\varphi')\exp(-\varphi^2/2)d\varphi \leq (r/\varphi'_m)\int_{\varphi(b)}^\infty (1/\varphi)\exp(-\varphi^2/2)d\varphi$, $\varphi'_m = \min_{b\leq\eta<\infty} \varphi'$; From the convergence of the last integral, which is easy to show, it follows that the integral of Eq. (13) converges.

NOTATION

u and v — are the longitudinal and transverse velocity components;
x and y — are the longitudinal and transverse coordinates;
ρ — is the density;

p	is the pressure;
T	is the temperature;
h	is the enthalpy;
V	is the specific volume;
Pr	is the Prandtl number;
z	is the dryness level;
$\varkappa$	is the adiabatic exponent of the heated vapor;
λ, μ, ν	are the coefficients of thermal conductivity, dynamic and kinematic viscosity, respectively.

The subscripts l and v refer, respectively, to parameters of the liquid and vapor phase; e refers to parameters of the external flow.

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DISCHARGE AND DYNAMIC CHARACTERISTICS OF BOILING WATER IN LAVAL NOZZLES

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The criterial processing of experimental results, obtained by studying the discharge of hot water into the atmosphere through a Laval nozzle, is proposed. The initial parameters of the water were varied over the range p_0 = 4.9-17.6 bar and t_0 = 119-204°C.

The effect is investigated in this paper of the initial parameters of hot water, p_0 and t_0 [$t_0 > t_s(p_n)$], where $t_s(p_n)$ is the saturation temperature at the temperature of the surrounding medium, on the discharge characteristics, the mean-mass efflux velocities, and pressures for nozzles of specified geometry.

The experiments were conducted in a facility representing a hydraulic circuit, including a high-pressure tank, the working section – the nozzle, and a catcher vessel. In order to produce water with specified initial parameters, a number of anciliary devices were provided: a heat-exchanger; electric heater; a compressed air supply line; and pumps.

The experiments were carried out with water that had not undergone preliminary deaeration and purification; however, during heating up in the closed volume, a periodic pressure "scouring" was carried out, which led to a reduction of the amount of gas in the water. The water obtained was underheated by 1-2° before the saturation line. Greater underheatings were achieved by the addition of compressed air.

During the experiment, the initial temperature of the water (t_0), the initial pressure (p_0), the static pressure along the nozzle (p_i), the water flow rate per second (G), and the reactive thrust (R) were measured. From the measured values of G, R, and p_0, the mean flow velocity at the outlet was calculated by the formula

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